ARE YOU READY FOR CALCULUS?

1. Simplify: (a) $\dfrac{x^3-9x}{x^2-7x+12}$ (b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$ (c) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ (d) $\dfrac{9-x^{-2}}{3+x^{-1}}$

2. Rationalize the denominator: (a) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$ (b) $\dfrac{4}{1-\sqrt{5}}$ (c) $\dfrac{1}{1+\sqrt{3}-\sqrt{5}}$

3. Write each of the following expressions in the form ca^pb^q where c, p and q are numbers:
   (a) $\dfrac{(2a^2)^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\dfrac{a(2/b)}{3/a}$ (d) $\dfrac{ab-a}{b^2-b}$ (e) $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$ (f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

4. Solve for x (do not use a calculator):
   (a) $5^{(x+1)}=25$ (b) $\dfrac{1}{3}=3^{2x+2}$ (c) $\log_2 x=3$ (d) $\log_3 x^2=2\log_3 4-4\log_3 5$

5. Simplify: (a) $\log_2 5+\log_2(x^2-1)-\log_2(x-1)$ (b) $2\log_4 9-\log_2 3$ (c) $3^{2\log_3 5}$

6. Simplify: (a) $\log_{10}\left(10^{1/2}\right)$ (b) $\log_{10}\left(\frac{1}{10^x}\right)$ (c) $2\log_{10}\sqrt{x}+3\log_{10}x^{1/3}$

7. Solve the following equations for the indicated variables:
   (a) $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$, for a (b) $V=2(ab+bc+ca)$, for a
   (c) $A=2\pi r^2+2\pi rh$, for positive r (d) $A=P+nrP$, for P
   (e) $2x-2yd=y+xd$, for d (f) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$, for x

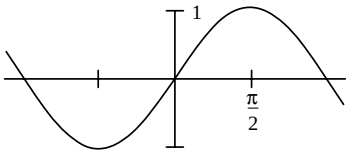
8. For the equations (a) $y=x^2+4x+3$ (b) $3x^2+3x+2y=0$ (c) $9y^2-6y-9-x=0$ complete the square and reduce to one of the standard forms $y-b=A(x-a)^2$ or $x-a=A(y-b)^2$.

9. Factor completely: (a) x^6-16x^4 (b) $4x^3-8x^2-25x+50$ (c) $8x^3+27$ (d) x^4-1

10. Find <u>all</u> real solutions to: (a) $x^6-16x^4=0$ (b) $4x^3-8x^2-25x+50=0$ (c) $8x^3+27=0$

11. Solve for x: (a) $3\sin^2 x=\cos^2 x\ ;\quad 0\le x<2\pi$ (b) $\cos^2 x-\sin^2 x=\sin x\ ;\quad -\pi<x\le\pi$
    (c) $\tan x+\sec x=2\cos x\ ;\qquad -\infty<x<\infty$

12. Without using a calculator, evaluate the following:
    (a) $\cos\ 210^\circ$ (b) $\sin\ \dfrac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$
    (e) $\cos\ \dfrac{9\pi}{4}$ (f) $\sin^{-1}\ \dfrac{\sqrt{3}}{2}$ (g) $\tan\ \dfrac{7\pi}{6}$ (h) $\cos^{-1}(-1)$



13. Given the graph of $\sin x$, sketch the graphs of:
    (a) $\sin\left(x-\dfrac{\pi}{4}\right)$ (b) $\sin\left(\dfrac{x}{2}\right)$ (c) $2\sin x$ (d) $\cos x$ (e) $\dfrac{1}{\sin x}$

14. Solve the equations: (a) $4x^2+12x+3=0$ (b) $2x+1=\dfrac{5}{x+2}$ (c) $\dfrac{x+1}{x}-\dfrac{x}{x+1}=0$

15. Find the remainders on division of:
    (a) $x^5-4x^4+x^3-7x+1$ by $x+2$. (b) $x^5-x^4+x^3+2x^2-x+4$ by x^3+1.

16. (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.
(b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$. (All solutions are rational and between ± 1.)

17. Solve the inequalities (a) $x^2 + 2x - 3 \leq 0$ (b) $\dfrac{2x-1}{3x-2} \leq 1$ (c) $x^2 + x + 1 > 0$

18. Solve for x: (a) $|-x+4| \leq 1$ (b) $|5x-2| = 8$ (c) $|2x+1| = x+3$

19. Determine the equations of the following lines: (a) the line through (-1,3) and (2,-4);
(b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$;
(c) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$.

20. (a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$
(b) Shade the region in the $x - y$ plane that is described by the inequalities $\begin{cases} 3x - y - 7 & < & 0 \\ x + 5y + 3 & \geq & 0 \end{cases}$.

21. Find the equations of the following circles:
(a) the circle with centre at $(1, 2)$ that passes through the point $(-2, -1)$;
(b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the $x-$ and $y-$ axes, respectively.

22. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:
(a) the centre and radius; (b) the equation of the tangent at $(-2, 5)$

23. A circle is tangent to the $y-$axis at $y = 3$ and has one $x-$intercept at $x = 1$.
(a) Determine the other $x-$intercept. (b) Deduce the equation of the circle.

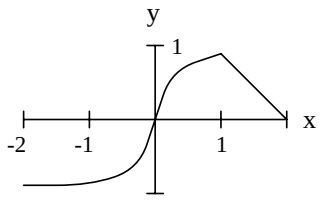
24. A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is three times its distance from the point $B(2, -1)$. Determine the equation of the curve.

25. (a) Find the domain of the function $f(x) = \dfrac{3x+1}{\sqrt{x^2+x-2}}$.
(b) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \dfrac{5x-3}{2x+1}$

26. Let $f(x) = \dfrac{|\,x\,|}{x}$. Show that $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$. Find the domain and range of $f(x)$.

27. Simplify $\dfrac{f(x+h) - f(x)}{h}$, where (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{1}{x+1}$ (c) $f(x) = x^2$.

28. The graph of the function $y = f(x)$ is given as follows:



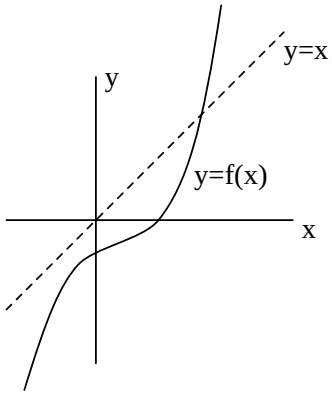
Determine the graphs of the functions:
(a) $f(x+1)$ (b) $f(-x)$ (c) $|f(x)|$ (d) $f(|x|)$

29. Sketch the graphs of the functions: (a) $g(x) = |3x + 2|$ (b) $h(x) = |x(x-1)|$

30. (a) The graph of a quadratic function (a parabola) has $x-$intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.
(b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$.

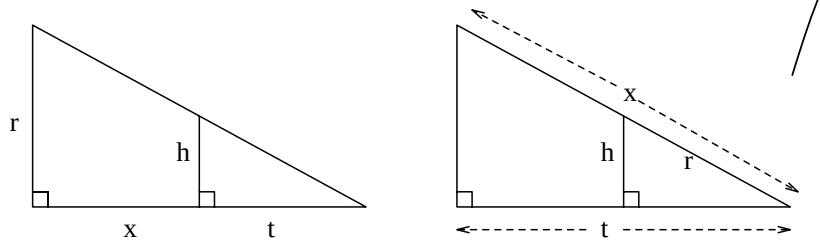
31. Write as a single equation in x and y: (a) $\begin{cases} x &= t+1 \\ y &= t^2 - t \end{cases}$ (b) $\begin{cases} x &= \sqrt[3]{t} - 1 \\ y &= t^2 - t \end{cases}$ (c) $\begin{cases} x &= \sin t \\ y &= \cos t \end{cases}$

32. Find the inverse of the functions: (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{x+2}{5x-1}$
(c) $f(x) = x^2 + 2x - 1\,, \quad x > 0$



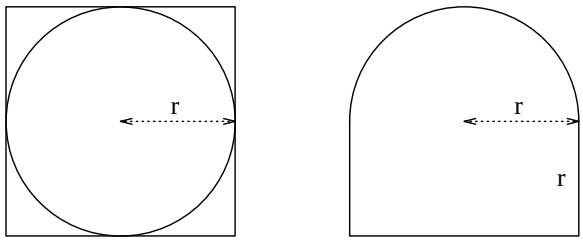
33. A function $f(x)$ has the graph to the right
Sketch the graph of the inverse function $f^{-1}(x)$.

34. Express x in terms of the other variables in the picture.



(a) (b)

35. (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a) (b)

(b) Find a formula for the perimeter of a window of the shape in the picture (b) above.

(c) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

(d) Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?

(e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

36. You should know the following trigonometric identities.

(A) $\sin(-x) = -\sin x$ (C) $\cos(x+y) = \cos x \cos y - \sin x \sin y$
(B) $\cos(-x) = \cos x$ (D) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

Use these to derive the following important identities, which you should also know.

(a) $\sin^2 x + \cos^2 x \equiv 1$ (use C and $\cos 0 = 1$) (b) $\sin 2x \equiv 2 \sin x \cos x$ (c) $\cos 2x \equiv \cos^2 x - \sin^2 x$

(d) $\cos 2x \equiv 2\cos^2 x - 1$ (e) $\cos 2x \equiv 1 - 2\sin^2 x$ (f) $\left|\cos \dfrac{x}{2}\right| \equiv \sqrt{\dfrac{1+\cos x}{2}}$ (g) $\left|\sin \dfrac{x}{2}\right| \equiv \sqrt{\dfrac{1-\cos x}{2}}$